الجامعة السورية الخاصة
SYRIAN PRIVATE UNIVERSITY

كـلـيــة هندسة الحاسوب والمعلوماتية والاتصالات
Faculty of Computer & Informatics
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Logic Circuits

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Chapter_ 4
Boolean Algebra
(الجبر البوليني)

Lecture _06
Boolean Operations & Laws
(القوانين والعمليات البولينية)

6-1. Boolean Operations and Expressions (التعابير والعمليات البولينية)

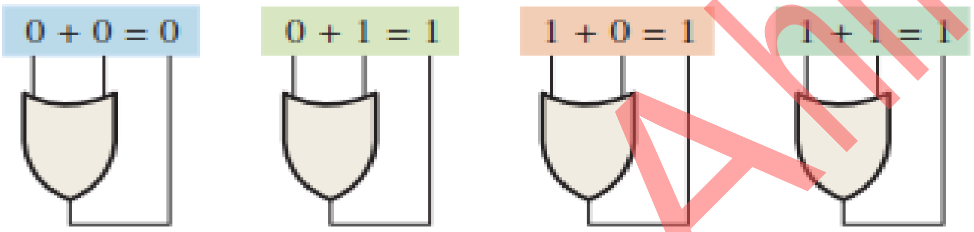
Variable, **complement** and **literal** are terms used in Boolean algebra.

- A **variable** (عنصر متغير) is a symbol used to represent an action, a condition, or data. A single variable can only have a value of 1 or 0.
- The **complement** (متمم) represents the inverse of a variable and is indicated with an overbar (خط فوق الرمز).

  Thus, the complement of A is $\overline{A}$.
- A **literal** (بيانات حرفية) is a variable or its complement.

Boolean Addition operation

- Addition is equivalent to the **OR** operation. The basic rules are shown in Fig.

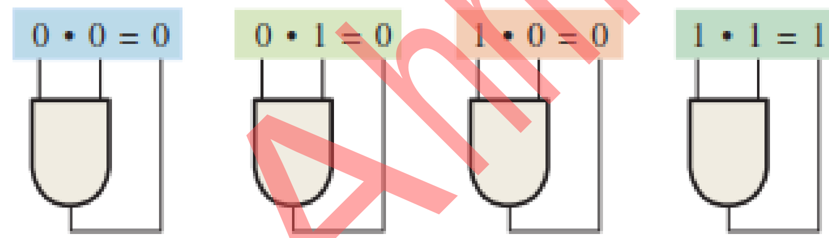


  - In Boolean algebra, a sum term is a sum of literals.
  - In logic circuits, a sum term is produced by an **OR** operation with **no AND** operations involved. Some examples of sum terms are

$$A+B;\ \ A+\overline{B};\ A+B+\overline{C};\ \ \overline{A}+B+C+\overline{D}$$

Boolean Multiplication operation

- In Boolean algebra, multiplication is equivalent to the **AND** operation. The basic rules are shown in Fig.



  - In Boolean algebra, a product term is the product of literals.
  - In logic circuits, a product term is produced by an **AND** operation with no **OR** operations involved.
  - Some examples of product terms are: AB; $A\overline{B}$; ABC; $A\overline{B}C\overline{D}$

Example 6-1 Determine the values of *A*, *B*, C and D that make the sum term of the expression $A+\overline{B}+C+\overline{D}=0$?

Solution For the sum term to be 0, each literal must = 0;

therefore *A* = 0, *B* =1 (so that $\overline{B}=0$), *C* = 0, and *D* =1 (so that $\overline{D}=0$).

$$A+\overline{B}+C+\overline{D}=0+\overline{1}+0+\overline{1}=0+0+0+0=0$$

****

Example 6-2 Determine the values of *A*, *B*, C and D that make the product term of the expression $A\overline{B}C\overline{D}=1$?

Solution For the sum term to be 1, each literal must = 1;

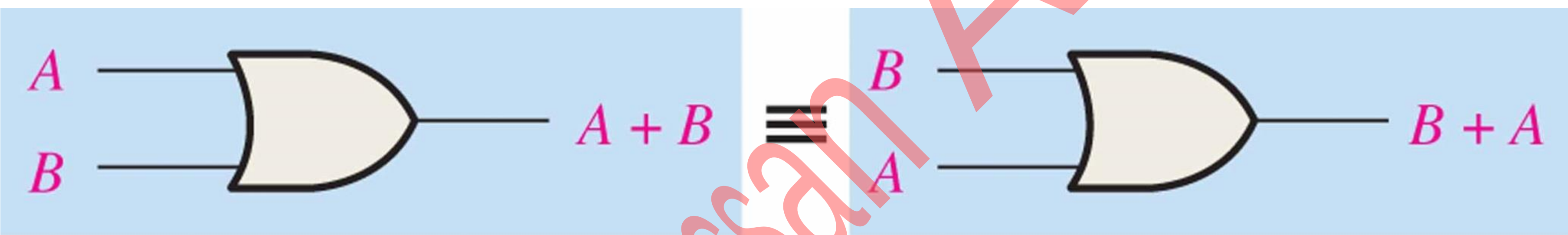
therefore *A* = 1, *B* =0 (so that $\overline{B}=1$), *C* =1, and *D*=0 (so that $\overline{D}=1$).

$$A\overline{B}C\overline{D}=1\cdot\overline{0}\cdot1\cdot\overline{0}=1\cdot1\cdot1\cdot1=1$$

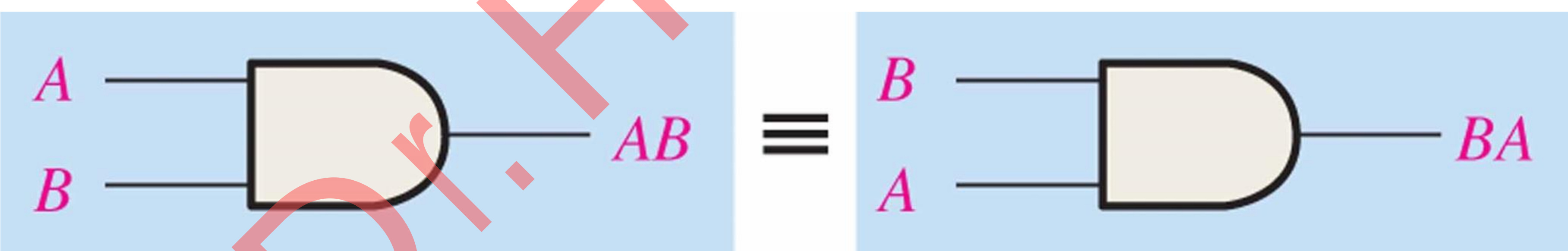
6-2. Laws of Boolean Algebra (قوانين الجبر البوليني)

Commutative Laws (قوانين التبادل)

- The commutative laws are applied to addition and multiplication.
  - For **addition**, the commutative law states $A + B = B + A$

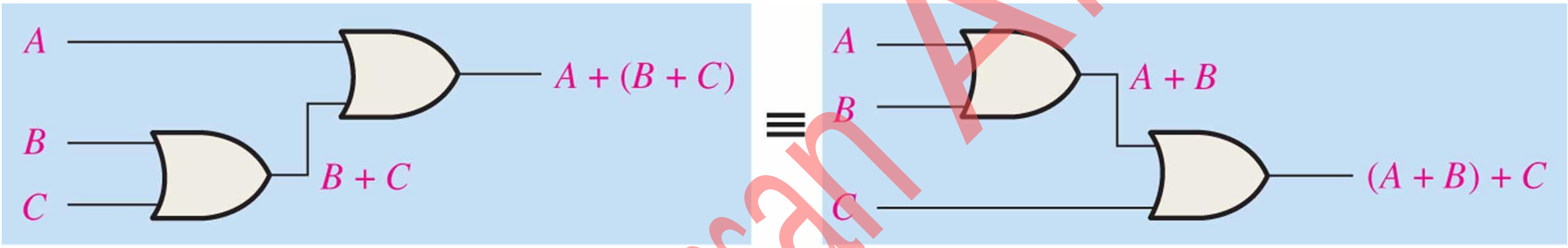


- For **multiplication**, the commutative law states $A \cdot B = B \cdot A$

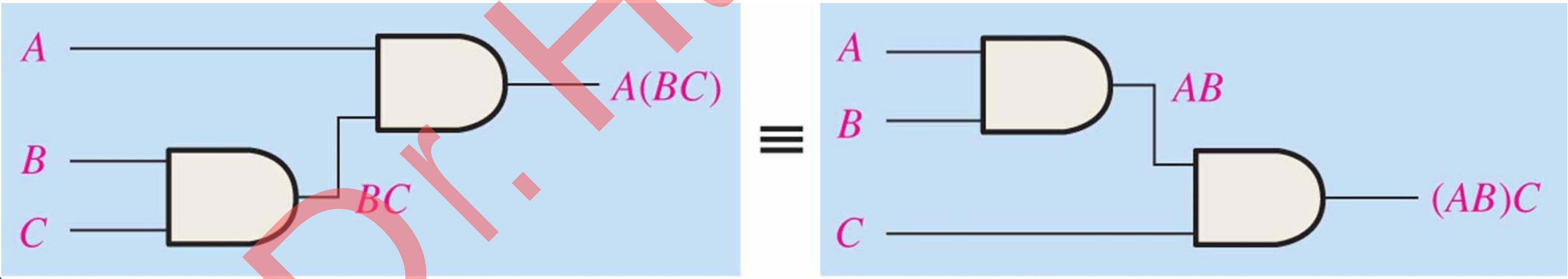


Associative Laws (قوانين التجميع)

- The associative laws are also applied to addition and multiplication.
  - For **addition**, the commutative law states $\boxed{A+(B+C)=(A+B)+C}$

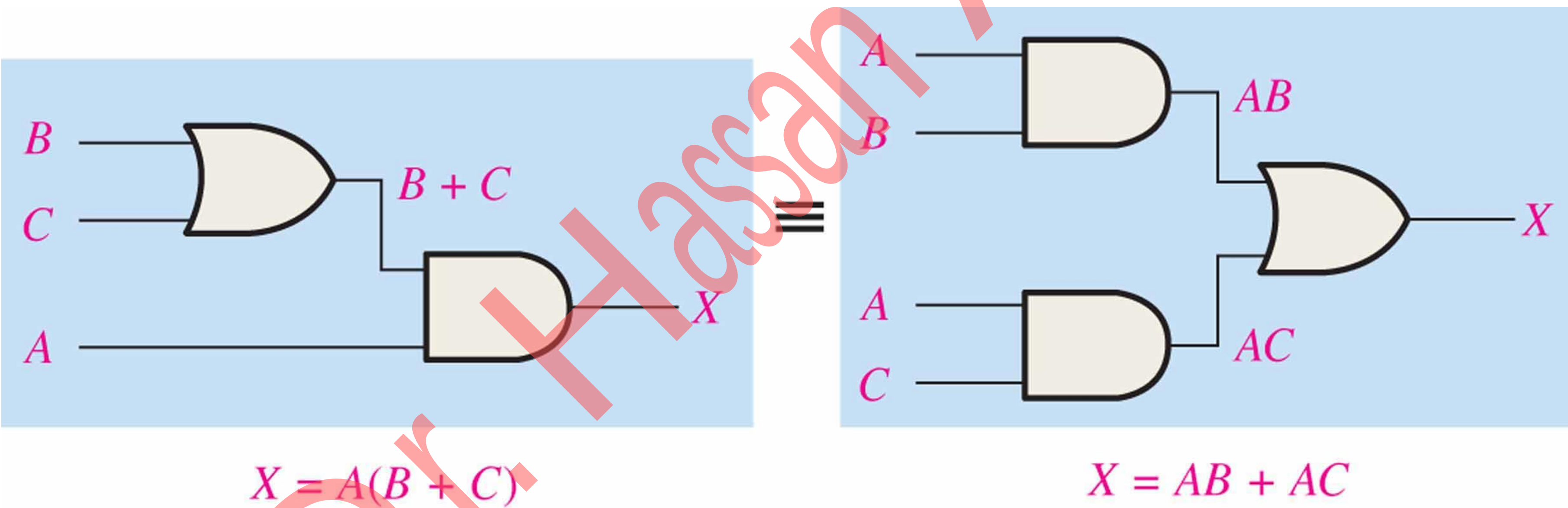


- For **multiplication**, the commutative law states $\boxed{A\cdot(B\cdot C)=(A\cdot B)\cdot C}$



Distributive Law (قانون التوزيع)

- The **distributive law** is the factoring law (قانون العوامل).
  - A common variable can be factored (محللة إلى عوامل) from an expression just as in ordinary algebra. That is $\boxed{A\cdot(B+C)=A\cdot B+A\cdot C}$



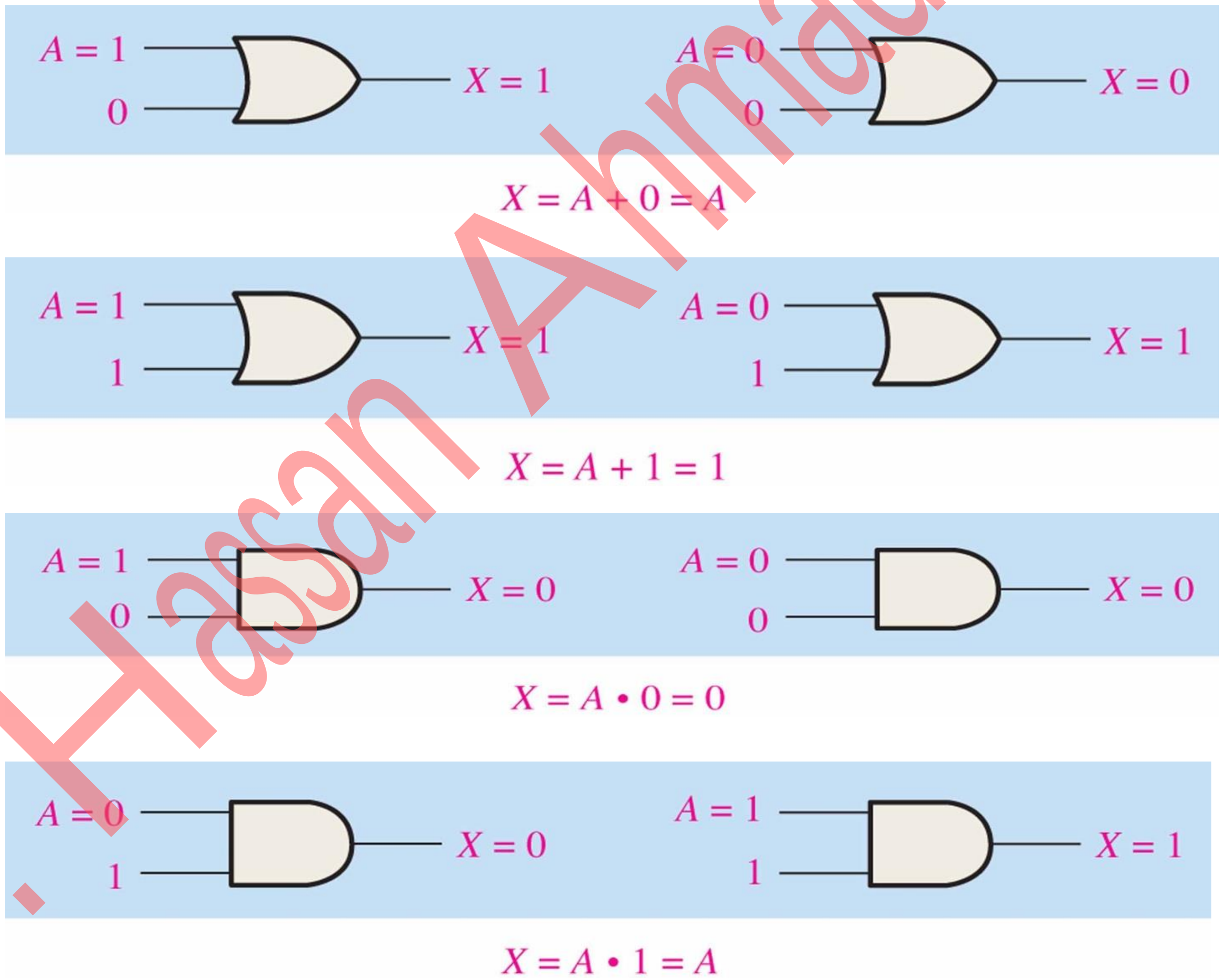
6-3. Rules of Boolean Algebra (قواعد الجبر البوليني)

Rule_1: $A + 0 = A$

Rule_2: $A + 1 = 1$

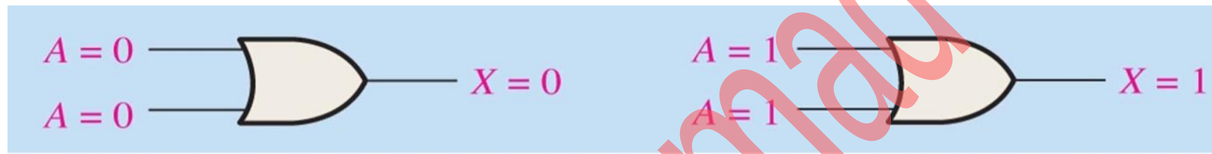
Rule_3: $A.0 = 0$

Rule_4: $A.1 = A$



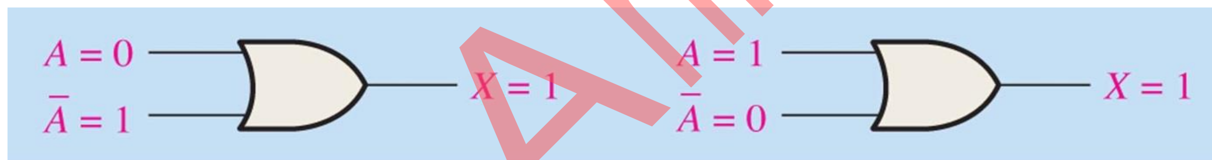
Rules of Boolean Algebra

Rule_5: $A + A = A$



$X = A + A = A$

Rule_6: $A + \overline{A} = 1$



$X = A + \overline{A} = 1$

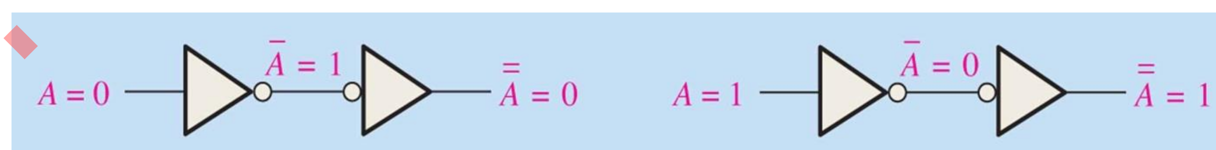
Rule_7: $A \cdot A = A$

$X = A \bullet A = A$

Rule_8: $A \cdot \overline{A} = 0$

$X = A \bullet \overline{A} = 0$

Rule_9: $\overline{\overline{A}} = A$



$\overline{\overline{A}} = A$

Rules of Boolean Algebra

Rule_10: $A + AB = A$

This rule can be proved by applying the distributed law, rule_2, and rule_4 as follows:

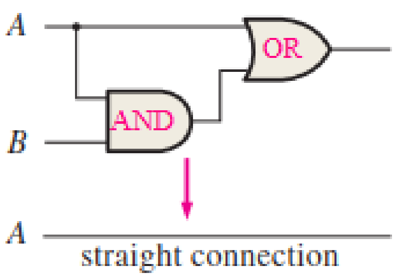
$$
\begin{aligned}
A + AB &= A(1+B) && \text{Distributive law} \\
&= A \cdot 1 && \text{Rul\_2}: (1+B) = 1 \\
&= A && \text{Rul\_4}: A \cdot 1 = A
\end{aligned}
$$

The truth table and resulting logic circuit simplification is

Rule 10: $A + AB = A$.

A	B	AB	$A + AB$
0	0	0	0
0	1	0	0
1	0	0	1
1	1	1	1

equal



Rules of Boolean Algebra

Rule_11: $A + \overline{A}B = A + B$

This rule can be proved as follows:

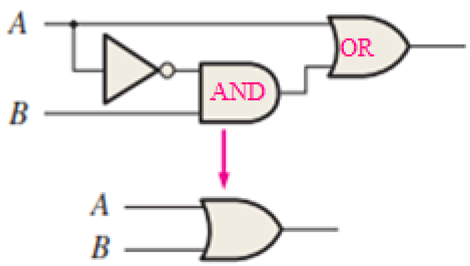
$$
\begin{aligned}
A + \overline{A}B &= (A + AB) + \overline{A}B && \text{Rul\_10: } A = A + AB \\
&= (AA + AB) + \overline{A}B && \text{Rul\_7: } A = AA \\
&= AA + AB + A\overline{A} + \overline{A}B && \text{Rul\_8: adding } A\overline{A} = 0 \\
&= (A + \overline{A})(A + B) && \text{Distributed law} \\
&= 1 \cdot (A + B) && \text{Rul\_6: } A + \overline{A} = 1 \\
&= A + B
\end{aligned}
$$

The truth table and resulting logic circuit simplification is

Rule 11: $A + \overline{A}B = A + B$.

A	B	$\overline{A}B$	$A + \overline{A}B$	$A + B$
0	0	0	0	0
0	1	1	1	1
1	0	0	1	1
1	1	0	1	1

equal



Rules of Boolean Algebra

Rule_12: $(A+B)(A+C) = A+BC$

This rule can be proved as follows:

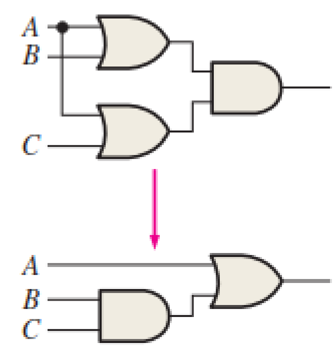
$$
\begin{aligned}
(A+B)(A+C) &= AA + AC + AB + BC && \text{Distributed law} \\
&= A + AC + AB + BC && \text{Rul\_7: } A = AA \\
&= A(1+C) + AB + BC && \text{Distributed law} \\
&= A\cdot 1 + AB + BC && \text{Rul\_2: } 1+C=1 \\
&= A(1+B) + BC && \text{Distributed law} \\
&= A\cdot 1 + BC && \text{Rul\_2: } 1+B=1 \\
&= A + BC && \text{Rul\_4: } A\cdot 1 = A
\end{aligned}
$$

The truth table and resulting logic circuit simplification is

Rule 12: $(A+B)(A+C) = A+BC$.

A	B	C	$A+B$	$A+C$	$(A+B)(A+C)$	BC	$A+BC$
0	0	0	0	0	0	0	0
0	0	1	0	1	0	0	0
0	1	0	1	0	0	0	0
0	1	1	1	1	1	1	1
1	0	0	1	1	1	0	1
1	0	1	1	1	1	0	1
1	1	0	1	1	1	0	1
1	1	1	1	1	1	1	1

equal



Basic rules of Boolean algebra.

1. $A + 0 = A$	**7.** $A \cdot A = A$
2. $A + 1 = 1$	**8.** $A \cdot \overline{A} = 0$
3. $A \cdot 0 = 0$	**9.** $\overline{\overline{A}} = A$
4. $A \cdot 1 = A$	**10.** $A + AB = A$
5. $A + A = A$	**11.** $A + \overline{A}B = A + B$
6. $A + \overline{A} = 1$	**12.** $(A + B)(A + C) = A + BC$

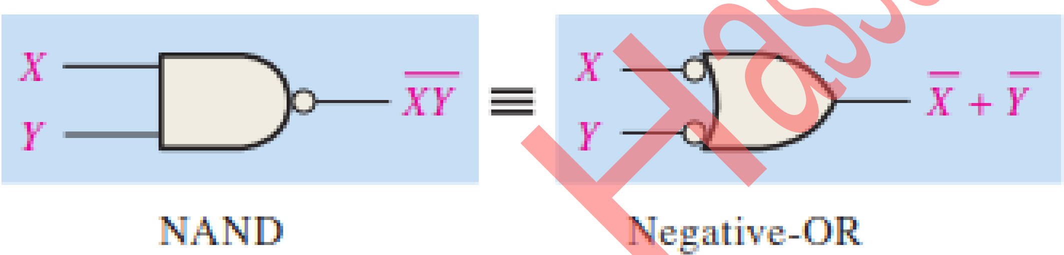
A, *B*, or *C* can represent a single variable or a combination of variables.

6-4. DeMorgan's Theorems

DeMorgan's first Theorem

DeMorgan's first theorem is stated as follows:

- **The complement of a product of variables is equal to the sum of the complements of the variables.**
- The formula for expressing this theorem for two variables is $\overline{XY} = \overline{X} + \overline{Y}$

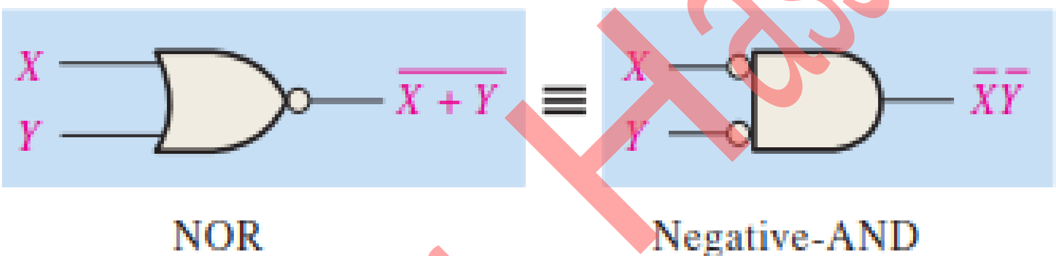


Inputs		Output	
X	Y	$\overline{XY}$	$\overline{X} + \overline{Y}$
0	0	1	1
0	1	1	1
1	0	1	1
1	1	0	0

DeMorgan's second Theorem

DeMorgan's second theorem is stated as follows:

- **The complement of a sum of variables is equal to the product of the complements of the variables.**
- The formula for expressing this theorem for two variables is $\overline{X+Y} = \overline{X} \cdot \overline{Y}$



Inputs		Output	
X	Y	$\overline{X+Y}$	$\overline{X}\,\overline{Y}$
0	0	1	1
0	1	0	0
1	0	0	0
1	1	0	0

Example 6-3 Applying DeMorgan's theorems to the expressions

$$\overline{XYZ}; \qquad \overline{X+Y+Z}$$

$$\overline{WXYZ}; \qquad \overline{W+X+Y+Z}$$

Solution

$$\overline{X\,Y\,Z} = \overline{X} + \overline{Y} + \overline{Z}$$

$$\overline{WX\,Y\,Z} = \overline{W} + \overline{X} + \overline{Y} + \overline{Z}$$

$$\overline{X+Y+Z} = \overline{X}\,\overline{Y}\,\overline{Z}$$

$$\overline{W+X+Y+Z} = \overline{W}\,\overline{X}\,\overline{Y}\,\overline{Z}$$

Applying DeMorgan's Theorems

The following procedure illustrates the application of DeMorgan's theorems and Boolean algebra to the specific expression $\overline{\overline{A+B\overline{C}}+D(\overline{E+\overline{F}})}$

Step 1: Identify the terms to which you can apply DeMorgan's theorems, and think of each term as a single variable. Let $\overline{A+B\overline{C}} = X$ and $D(\overline{E+\overline{F}}) = Y$.

Step 2: Since $\overline{X+Y} = \overline{X}\,\overline{Y}$,

$$\overline{(\overline{A+B\overline{C}})+(D(\overline{E+\overline{F}}))} = (\overline{\overline{A+B\overline{C}}})(\overline{D(\overline{E+\overline{F}})})$$

Step 3: Use rule 9 ($\overline{\overline{A}} = A$) to cancel the double bars over the left term (this is not part of DeMorgan's theorem).

$$(\overline{\overline{A+B\overline{C}}})(\overline{D(\overline{E+\overline{F}})}) = (A+B\overline{C})(\overline{D(\overline{E+\overline{F}})})$$

Step 4: Apply DeMorgan's theorem to the second term.

$$(A+B\overline{C})(\overline{D(\overline{E+\overline{F}})}) = (A+B\overline{C})(\overline{D}+(\overline{\overline{E+\overline{F}}}))$$

Step 5: Use rule 9 ($\overline{\overline{A}} = A$) to cancel the double bars over the $E+\overline{F}$ part of the term.

$$(A+B\overline{C})(\overline{D}+\overline{\overline{E+\overline{F}}}) = (A+B\overline{C})(\overline{D}+E+\overline{F})$$

Example 6-4

Apply DeMorgan's theorems to each of the following expressions:

(a) $\overline{(A + B + C)D}$ (b) $\overline{ABC + DEF}$ (c) $\overline{A\overline{B} + \overline{C}D + EF}$

Solution

(a) Let $A + B + C = X$ and $D = Y$. The expression $\overline{(A + B + C)D}$ is of the form $\overline{XY} = \overline{X} + \overline{Y}$ and can be rewritten as

$$\overline{(A + B + C)D} = \overline{A + B + C} + \overline{D}$$

Next, apply DeMorgan's theorem to the term $\overline{A + B + C}$.

$$\overline{A + B + C} + \overline{D} = \overline{A}\,\overline{B}\,\overline{C} + \overline{D}$$

(b) Let $ABC = X$ and $DEF = Y$. The expression $\overline{ABC + DEF}$ is of the form $\overline{X + Y} = \overline{X}\,\overline{Y}$ and can be rewritten as

$$\overline{ABC + DEF} = (\overline{ABC})(\overline{DEF})$$

Next, apply DeMorgan's theorem to each of the terms $\overline{ABC}$ and $\overline{DEF}$.

$$(\overline{ABC})(\overline{DEF}) = (\overline{A} + \overline{B} + \overline{C})(\overline{D} + \overline{E} + \overline{F})$$

(c) Let $A\overline{B} = X$, $\overline{C}D = Y$, and $EF = Z$. The expression $\overline{A\overline{B} + \overline{C}D + EF}$ is of the form $\overline{X + Y + Z} = \overline{X}\,\overline{Y}\,\overline{Z}$ and can be rewritten as

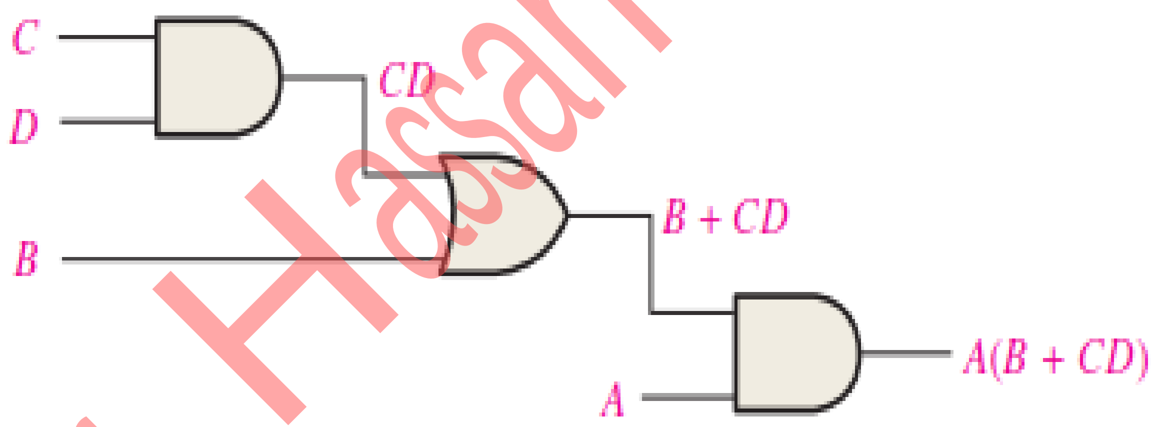
$$\overline{A\overline{B} + \overline{C}D + EF} = (\overline{A\overline{B}})(\overline{\overline{C}D})(\overline{EF})$$

Next, apply DeMorgan's theorem to each of the terms $\overline{A\overline{B}}$, $\overline{\overline{C}D}$, and $\overline{EF}$.

$$(\overline{A\overline{B}})(\overline{\overline{C}D})(\overline{EF}) = (\overline{A} + B)(C + \overline{D})(\overline{E} + \overline{F})$$

6-5. Boolean Analysis of Logic Circuits

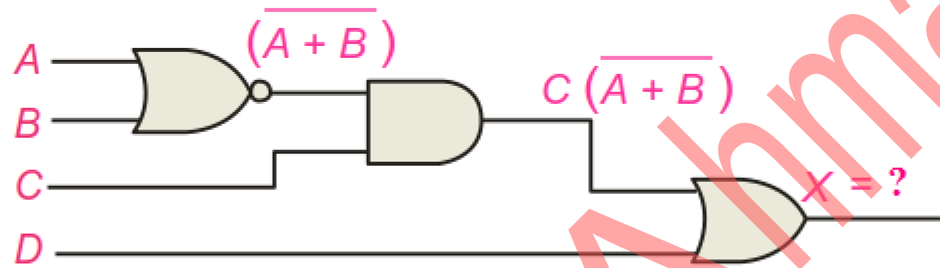
- Combinational (التوافقية) logic circuits can be analyzed by writing the expression for each gate and combining the expressions according to the rules for Boolean algebra.
- For the example, circuit in Fig.



Therefore, the expression for this AND gate is *A*(*B* + *CD*), which is the final output expression for the entire circuit.

Example 6-5

Given the logic circuit. Apply Boolean algebra to derive the expression for X.



Solution

Write the expression for each gate:

1) For NOR (NOT-OR) gate with A and B inputs we have: $\overline{A+B}$

2) For AND gate with $\overline{A+B}$ and C inputs we have: $C(\overline{A+B})$

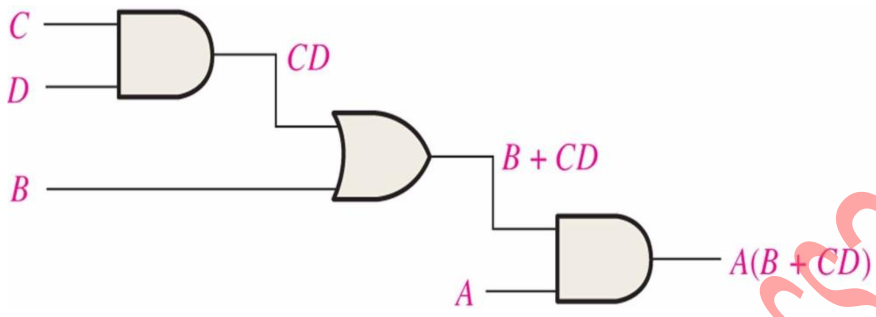
3) For OR gate with $C(\overline{A+B})$ and D inputs we have: $C(\overline{A+B})+D$

Therefore, $X = C(\overline{A+B})+D$

Applying DeMorgan's theorems and the distribution law:

$$\overline{A+B} = \overline{A}\cdot\overline{B} \Rightarrow X = C(\overline{A}\cdot\overline{B})+D = \overline{A}\cdot\overline{B}\cdot C + D$$

Constructing a Truth Table for a Logic Circuit



Inputs				Output
A	B	C	D	$A(B + CD)$
0	0	0	0	0
0	0	0	1	0
0	0	1	0	0
0	0	1	1	0
0	1	0	0	0
0	1	0	1	0
0	1	1	0	0
0	1	1	1	0
1	0	0	0	0
1	0	0	1	0
1	0	1	0	0
1	0	1	1	1
1	1	0	0	1
1	1	0	1	1
1	1	1	0	1
1	1	1	1	1

6-6. Logic Simplification Using Boolean Algebra (التبسيط المنطقي)

A simplified Boolean expression uses the fewest gates possible to implement a given expression.

Example 6-6 Using Boolean algebra techniques, simplify this expression:

$$AB + A(B + C) + B(B + C)$$

Solution

Step 1: Apply the distributive law to the second and third terms in the expression, as follows:

$$AB + AB + AC + BB + BC$$

Step 2: Apply rule 7 ($BB = B$) to the fourth term.

$$AB + AB + AC + B + BC$$

Step 3: Apply rule 5 ($AB + AB = AB$) to the first two terms.

$$AB + AC + B + BC$$

Step 4: Apply rule 10 ($B + BC = B$) to the last two terms.

$$AB + AC + B$$

Step 5: Apply rule 10 ($AB + B = B$) to the first and third terms.

$$B + AC$$

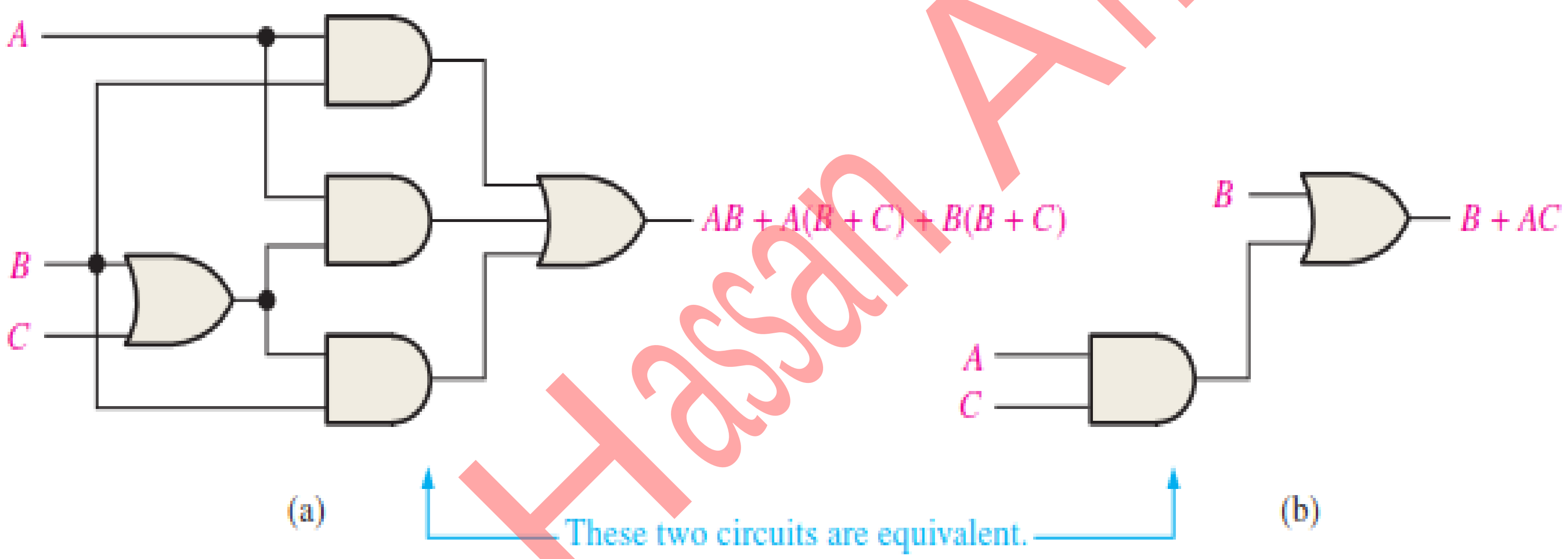
Basic rules of Boolean algebra.

1. $A + 0 = A$	**7.** $A \cdot A = A$
2. $A + 1 = 1$	**8.** $A \cdot \overline{A} = 0$
3. $A \cdot 0 = 0$	**9.** $\overline{\overline{A}} = A$
4. $A \cdot 1 = A$	**10.** $A + AB = A$
5. $A + A = A$	**11.** $A + \overline{A}B = A + B$
6. $A + \overline{A} = 1$	**12.** $(A + B)(A + C) = A + BC$

A, B, or C can represent a single variable or a combination of variables.

$$AB + A(B + C) + B(B + C) \equiv B + AC$$

The simplified circuit



Example 6-7

Simplify the following Boolean expression:

$$\overline{A}BC + A\overline{B}\,\overline{C} + \overline{A}\,\overline{B}\,\overline{C} + A\overline{B}C + ABC$$

Solution

Step 1: Factor BC out of the first and last terms.

$$BC(\overline{A} + A) + A\overline{B}\,\overline{C} + \overline{A}\,\overline{B}\,\overline{C} + A\overline{B}C$$

Step 2: Apply rule 6 ($\overline{A} + A = 1$) to the term in parentheses, and factor $A\overline{B}$ from the second and last terms.

$$BC \cdot 1 + A\overline{B}(\overline{C} + C) + \overline{A}\,\overline{B}\,\overline{C}$$

Step 3: Apply rule 4 (drop the 1) to the first term and rule 6 ($\overline{C} + C = 1$) to the term in parentheses.

$$BC + A\overline{B} \cdot 1 + \overline{A}\,\overline{B}\,\overline{C}$$

Step 4: Apply rule 4 (drop the 1) to the second term.

$$BC + A\overline{B} + \overline{A}\,\overline{B}\,\overline{C}$$

Step 5: Factor $\overline{B}$ from the second and third terms.

$$BC + \overline{B}(A + \overline{A}\,\overline{C})$$

Step 6: Apply rule 11 ($A + \overline{A}\,\overline{C} = A + \overline{C}$) to the term in parentheses.

$$BC + \overline{B}(A + \overline{C})$$

Step 7: Use the distributive and commutative laws to get the following expression:

$$BC + A\overline{B} + \overline{B}\,\overline{C}$$

6-6. Standard Forms of Boolean Expressions
(الصيغ النموذجية/ المعيارية للتعابير البولينية)

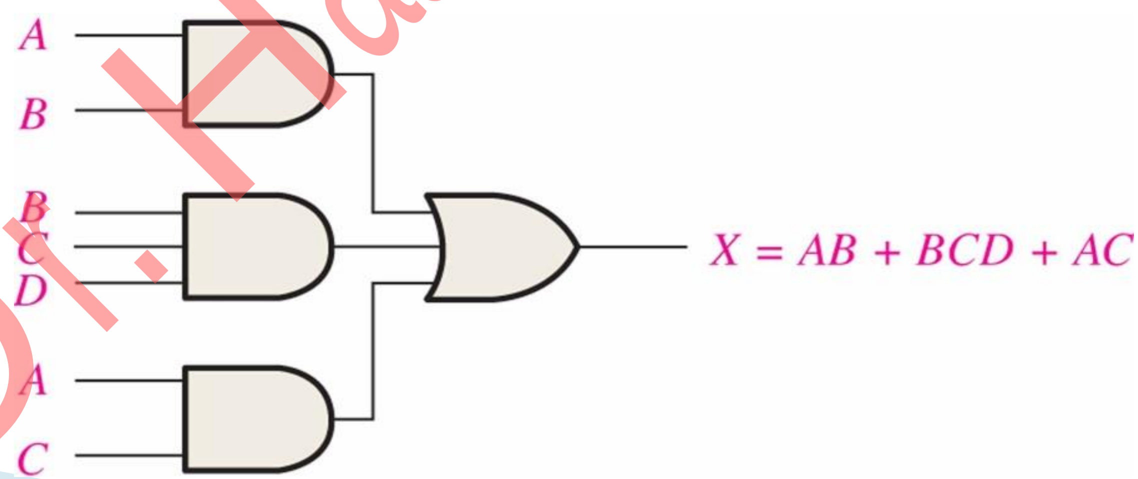
The Sum-of-Products (SOP) Form (جمع الجداءات)

- When two or more product terms are summed by Boolean addition, the resulting expression is a **sum-of-products (SOP)**.

Examples: $AB + ABC$; $ABC + CDE + \overline{B}C\overline{D}$; $\overline{A}B + \overline{A}B\overline{C} + AC$

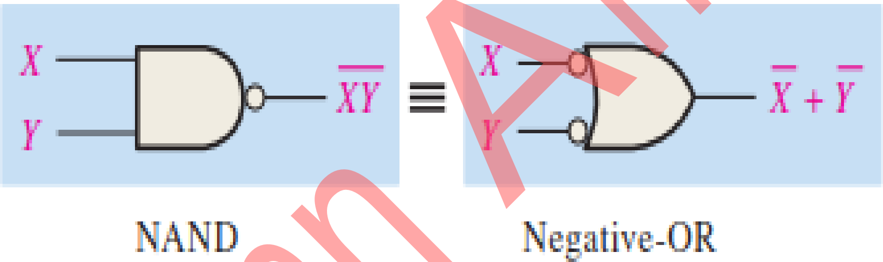
AND/OR Implementation of an SOP Expression:

$$AB + BCD + AC$$

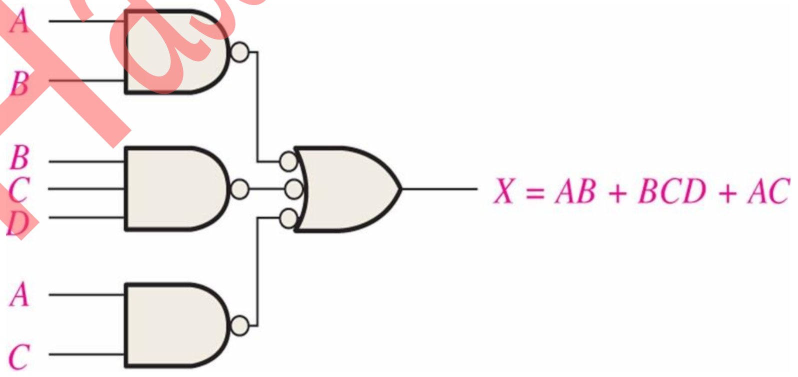


NAND/NAND Implementation of an SOP Expression:

- By using only NAND gates, an AND/OR function can be accomplished (=يُنجز يتحقق), as illustrated in Figure.
- The first level of NAND gates feed into (يغذي) a NAND gate that acts as a negative-OR gate.



- The NAND and negative-OR inversions cancel and the result is effectively an **AND/OR** circuit.



Conversion of General Expression to SOP Form

- Any logic expression can be changed into SOP form by applying Boolean algebra techniques.

For example, the expression $A(B+CD)$ can be converted to SOP form by applying the distributive law: $A(B+CD)$ = AB + ACD

Example 6-8 Convert each of the following Boolean expressions to SOP form:

$$AB + B(CD + EF); \quad (A+B)(B+C+D); \quad \overline{(\overline{A+B}) + C}$$

Solution

$$AB + B(CD + EF) = AB + BCD + BEF;$$

$$(A+B)(B+C+D) = AB + AC + AD + BB + BC + BD;$$

$$\overline{(\overline{A+B}) + C} = \overline{(\overline{A+B})}\,\overline{C} = (A+B)\overline{C} = A\overline{C} + B\overline{C}$$

SOP Standard form

- In **SOP standard form**, every variable in the domain must appear in each term. Another state is called **nonstandard form.**

For example:

Standard form: $A\overline{B}CD+\overline{A}\,\overline{B}C\overline{D}+ABC\overline{C}\,\overline{D}$

Nonstandard form: $A\overline{B}C+\overline{A}\,\overline{B}\,\overline{D}+AB\overline{C}\,\overline{D}$

Converting Product Terms to Standard SOP

- A nonstandard SOP expression can be converted into standard form using Boolean algebra rule 6 ($A+\overline{A}=1$).

Step 1: **Multiply** each nonstandard product term by a term made up of the sum of a missing variable (المتغيرة الضائعة) and its complement.

Step 2: Repeat Step 1 until all resulting product terms contain all variables in the domain in either complemented or uncomplemented form.

Example 6-9

Convert the following Boolean expression into standard SOP form:

$$A\overline{B}C + \overline{A}\,\overline{B} + AB\overline{C}D$$

Solution

The first term, $A\overline{B}C$, is missing variable D or $\overline{D}$, so multiply the first term by $D+\overline{D}$ as follows:

$$A\overline{B}C = A\overline{B}C(D+\overline{D}) = A\overline{B}CD + A\overline{B}C\overline{D}$$

The second term, $\overline{A}\,\overline{B}$, is missing variables C or $\overline{C}$ and D or $\overline{D}$, so first multiply the second term by $C+\overline{C}$ as follows:

$$\overline{A}\,\overline{B} = \overline{A}\,\overline{B}(C+\overline{C}) = \overline{A}\,\overline{B}C + \overline{A}\,\overline{B}\,\overline{C}$$

The two resulting terms are missing variable D or $\overline{D}$, so multiply both terms by $D+\overline{D}$ as follows:

$$\overline{A}\,\overline{B} = \overline{A}\,\overline{B}C + \overline{A}\,\overline{B}\,\overline{C} = \overline{A}\,\overline{B}C(D+\overline{D}) + \overline{A}\,\overline{B}\,\overline{C}(D+\overline{D})$$

$$= \overline{A}\,\overline{B}CD + \overline{A}\,\overline{B}C\overline{D} + \overline{A}\,\overline{B}\,\overline{C}D + \overline{A}\,\overline{B}\,\overline{C}\,\overline{D}$$

The third term, $AB\overline{C}D$, is already in standard form.

The complete standard SOP form of the original expression is as follows:

$$A\overline{B}CD + A\overline{B}C\overline{D} + \overline{A}\,\overline{B}CD + \overline{A}\,\overline{B}C\overline{D} + \overline{A}\,\overline{B}\,\overline{C}D + \overline{A}\,\overline{B}\,\overline{C}\,\overline{D} + AB\overline{C}D$$

The Product-of-Sums (POS) Form (جداء المجاميع)

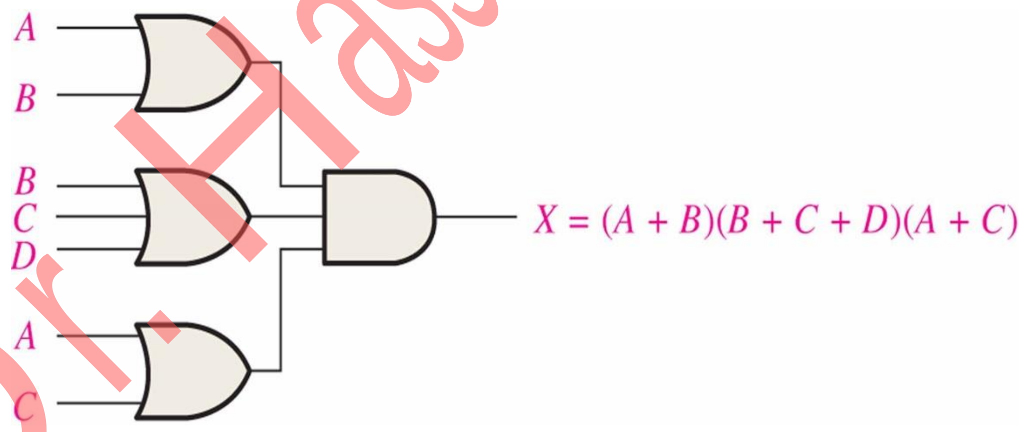
- When two or more sum terms are multiplied, the resulting expression is a **product-of-sums (POS)**.

Examples

$$(\overline{A}+B)(A+\overline{B}+C); \quad (\overline{A}+\overline{B}+\overline{C})(C+\overline{D}+E)(\overline{B}+C+D);$$

$$(A+B)(A+\overline{B}+C)(\overline{A}+C);$$

Implementation of a POS Expression: $(A+B)(B+C+D)(A+C)$



Standard Form of POS

- In **POS standard form**, every variable in the domain must appear in each sum term of the expression. Another state is called **nonstandard** form.
- For example,

Standard form: $(\overline{A}+\overline{B}+\overline{C}+\overline{D})(A+\overline{B}+C+D)(A+B+\overline{C}+D)$

Nonstandard form: $(\overline{A}+\overline{B}+C)(A+B+\overline{D})(A+\overline{B}+\overline{C}+D)$

Converting a Sum Term to Standard POS

- A nonstandard POS expression is converted into standard form using Boolean algebra rule 8 ($A\cdot\overline{A}=0$).

Step 1: **Add** to each nonstandard product term a term made up of the product of the missing variable and its complement.

Step 2: Apply rule 12 : $A + BC = (A + B)(A + C)$.

Step 3: Repeat Step 1 until all resulting sum terms contain all variables in the domain in either complemented or uncomplemented form.

Example 6-10

Convert the following Boolean expression into standard POS form: $(A+\overline{B}+C)(\overline{B}+C+\overline{D})(A+\overline{B}+\overline{C}+D)$

Solution

The first term, $(A+\overline{B}+C)$, is missing variable D or $\overline{D}$, so add $D\overline{D}$ and apply **rule 12** as follows: $A+\overline{B}+C+D\overline{D}=(A+\overline{B}+C+D)(A+\overline{B}+C+\overline{D})$

The second term, $(\overline{B}+C+\overline{D})$, is missing variable A or $\overline{A}$, so add $A\overline{A}$ and apply **rule 12** as follows: $\overline{B}+C+\overline{D}+A\overline{A}=(A+\overline{B}+C+\overline{D})(\overline{A}+\overline{B}+C+\overline{D})$

The third term, $A+\overline{B}+\overline{C}+D$, is already in standard form.

The standard POS form of the original expression is as follows:

$$(A+\overline{B}+C)(\overline{B}+C+\overline{D})(A+\overline{B}+\overline{C}+D)=$$

$$(A+\overline{B}+C+D)(A+\overline{B}+C+\overline{D})(A+\overline{B}+C+\overline{D})(\overline{A}+\overline{B}+C+\overline{D})(A+\overline{B}+\overline{C}+D)$$

Converting SOP Expressions to Truth Table Format

- The first step in constructing a truth table is to list all possible combinations of binary values of the variables in the expression.
- Next, convert the SOP expression to standard form if it is not already.
- Finally, place a 1 in the output column (***X***) for each binary value that makes the standard SOP expression **equal to 1** and place a **0** for all the remaining (المتبقية) binary values.

This procedure is illustrated in Example 6-11.

Example 6-11

Develop a truth table for the standard SOP expression

$$\overline{A}\,\overline{B}C + A\overline{B}\,\overline{C} + ABC$$

Solution

The binary values that make the product terms in the expressions equal to 1 are

$$\overline{A}\,\overline{B}C: 001; \quad A\overline{B}\,\overline{C}: 100; \quad ABC: 111$$

For each of these binary values, place a 1 in the output column as shown in the table. For each of the remaining binary combinations, place a 0 in the output column.

Inputs			Output	
A	*B*	*C*	*X*	Product Term
0	0	0	0	
0	0	1	1	$\overline{A}\,\overline{B}C$
0	1	0	0	
0	1	1	0	
1	0	0	1	$A\overline{B}\,\overline{C}$
1	0	1	0	
1	1	0	0	
1	1	1	1	ABC

Converting POS Expressions to Truth Table Format

To construct a truth table from a POS expression,

- First, list all the possible combinations of binary values of the variables just as was done for the SOP expression.
- Next, convert the POS expression to standard form if it is not already.
- Finally, place a 0 in the output column (***X***) for each binary value that makes the expression **equal to 0** and place a 1 for all the remaining binary values.

This procedure is illustrated in Example 6–12.

Example 6-12

Determine the truth table for the following standard POS expression:

$$(A+B+C)(A+\overline{B}+C)(A+\overline{B}+\overline{C})(\overline{A}+B+\overline{C})(\overline{A}+\overline{B}+C)$$

Solution

The binary values that make the sum terms in the expression equal to 0 are

$$A+B+C:000;\quad A+\overline{B}+C:010;\quad A+\overline{B}+\overline{C}:011;\quad \overline{A}+B+\overline{C}:101;\quad \overline{A}+\overline{B}+C:110;$$

For each of these binary values, place a 0 in the output column as shown in the table. For each of the remaining binary combinations, place a 1 in the output column.

Inputs			Output	
A	B	C	X	Sum Term
0	0	0	0	$(A+B+C)$
0	0	1	1	
0	1	0	0	$(A+\overline{B}+C)$
0	1	1	0	$(A+\overline{B}+\overline{C})$
1	0	0	1	
1	0	1	0	$(\overline{A}+B+\overline{C})$
1	1	0	0	$(\overline{A}+\overline{B}+C)$
1	1	1	1	

The end of Lecture_06, chapter 4